

A Distance-Independent Tree Model for Exotic Conifer Plantations in East Africa

DENIS ALDER

ABSTRACT. A simulation model of diameter increment and thinning for *Cupressus lusitanica*, *Pinus patula*, and *Pinus radiata* growing in Kenya, Tanzania, Uganda, and Malawi is presented. The diameter distribution is defined by a vector of diameters corresponding to fixed cumulative probability points. Increment of the diameter vector is a function of basal area relative to maximum basal area, diameter relative to dominant diameter, and height as a site-dependent index of age. A general algorithm for simulating thinning of the cumulative distribution is developed based upon the probability that a given stem will remain after thinning. Validation studies using permanent sample plot data for the 3 species show a standard deviation of percentage residuals for total volume prediction of 12-19 percent; they also show, through simulation of spacing experiments, that the model is accurate over a range of initial stockings from 120 to 1,400 stems/hectare. FOREST SCI. 25:59-71.

ADDITIONAL KEY WORDS. Diameter distribution, maximum basal area, dominance.

DISTANCE-INDEPENDENT TREE MODELS, in the terminology of Munro (1974), are those which are based upon a vector of tree diameters or diameter classes; but where the competitive stress of each tree or class does not require positional information about the trees. Such models assume therefore a close correlation between tree and stand variables and tree competitive status. The method is normally applied to the simulation of growth and yield in plantations which are even-aged and monospecific. Published examples are Clutter and Allison (1974), Gibson and others (1969), and Opie (1972).

In this work, a method is presented for modelling tree growth in which the vector of tree diameters is represented by tabulated points of a cumulative diameter distribution. It arose from the need to develop a fast, efficient, and compact model which would still provide detailed diameter distribution information, which could be adapted to different types of thinnings, and which would respond accurately over a very wide range of treatment. The result has been the emergence of the conceptual advances which are presented in this paper.¹

The data used to determine the empirical species-dependent coefficients of the model were derived from permanent sample plots in exotic coniferous plantations in Kenya, Tanzania, Uganda, and Malawi. So far, parameters have been determined for three species—*Pinus patula* Schlect & Cham., *Pinus radiata* D. Don,

¹ Other aspects of the model are described in "A growth and management model for coniferous plantations in East Africa," D. Alder, 1977. Unpublished Ph.D. thesis, Oxford Univ. 97 p.

The author is a Research Officer at the Unit of Tropical Silviculture, Department of Forestry, Oxford University, South Parks Road, Oxford OX1 3RB, UK. The work was financed by the United Kingdom Ministry of Overseas Development. The author gratefully acknowledges the essential assistance of Valerie J. Bowman, who was responsible for the figures and bibliographic research. He also wishes to thank Mrs. Hilda Pengelly for typing the manuscript. Manuscript received 7 March 1978.

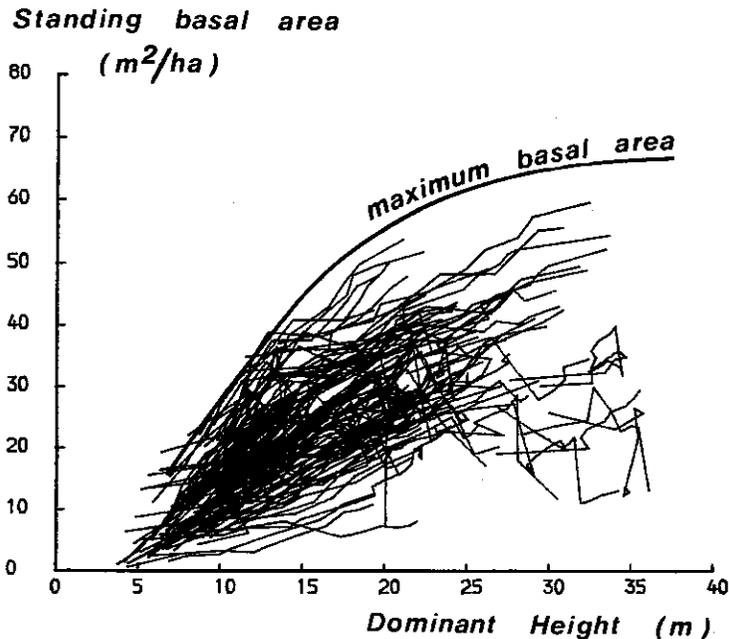


FIGURE 1. Hand-drawn maximum basal area line for *Pinus patula* permanent sample plots in Kenya.

and *Cupressus lusitanica* Mill. These three together form industrial plantations of approximately 200,000 ha in Kenya, Tanzania, and Malawi, and provide the basis for three distinct pulpwood schemes.

METHODS

Within the model, the distribution of diameters was represented by a vector of diameter values such that each corresponded to a distinct cumulative probability that that diameter would not be exceeded. Thus we have—

$$D = [d_1, d_2 \dots d_n]$$

$$P = [p_1, p_2 \dots p_n]$$

so that there is a probability p_i that any diameter sampled at random from the stand will be less than d_i . The total number, n , of elements in the vectors D and P , represents the discrimination of the model, and has been set to 10 in the current formulation. The probabilities of the p_i can be determined from their subscript and the value of n by

$$p_i = (i - 1/2)/n, \quad (1)$$

which with n as 10 gives implicit values for the p_i of

$$P = [0.05, 0.15, 0.25 \dots 0.85, 0.95].$$

If diameter increment increases monotonically with diameter for a given stand, the ranking of trees within the cumulative distribution is unaltered by increment. In this case, a function can be applied to the elements of the cumulative distribution vector D to estimate stand growth as if the elements were individual tree diameters.

The prediction of increment for a given diameter requires that three main effects should be taken into account. These are site, age, and competitive status. In practice, these factors are complex and composed of a number of subcomponents. In this model, the effect of site and age were largely unified into a single metric

TABLE 1. Coefficients for the limiting basal area function (see equation (2) in text).

Coefficient	<i>P. patula</i>	<i>P. radiata</i>	<i>Cupressus</i> sp.
<i>a</i>	67.8	86.3	63.9
<i>b</i>	0.1292	0.03382	0.1219
<i>c</i>	2.597	1.198	2.551

Note: Criteria of goodness of fit are not quoted as the function is based upon a hand-drawn curve enveloping permanent plot standing basal area data (see text), and hence such criteria would be meaningless in this context.

by the use of stand dominant height as a basic predictor variable for increment. Dominant height is itself determined within the model by a site- and age-dependent relationship which will not be discussed here.

The competitive status of a stand as whole can be considered as the ratio between the current basal area of the stand and the maximum basal area occurring at that site and age. In this model, little empirical information was available on maximum basal area as few older plots occurred at higher stockings. However, speculative hand-drawn curves which enveloped the maximum observed basal area values for a given species were tabulated against dominant height. An example, for *Pinus patula*, is shown in Figure 1. The Richards generalized growth function (cf. Pienaar and Turnbull 1973) was found to provide a suitable fit to points taken from this hand-drawn maximum basal area line. The function is

$$G_{\max} = a \cdot (1 - \exp[-b \cdot H])^c \quad (2)$$

where G_{\max} is maximum basal area, and a , b , and c are species-dependent empirical coefficients, whose values, fitted by nonlinear least squares, are shown in Table 1.

A measure of relative basal area q was defined such that at the maximum basal area, q was zero; whilst below the maximum it was positive, tending asymptotically to 1 as the stocking approached zero. Thus

$$q = 1 - G/G_{\max} \quad (3)$$

where G is the standing basal area. The relative basal area defines the potential for growth of the stand as a whole. It is obvious that a lower value of basal area on a given stand implies higher q . Less obviously, a lower dominant height at a given basal area means a smaller G_{\max} and hence lower q , and lower growth potential. In this way, q becomes dependent on site, as reflected in height growth at a given age; at a given basal area, a poorer site will have a lower G_{\max} and hence lower q .

For a single stand at a given relative basal area, the differences in competitive status among trees can be largely accounted for in terms of the ratio of the diameter of the tree to the mean diameter of the dominant trees. This is termed the dominance ratio r , by definition:

$$r = d_i/D_{\text{dom}} \quad (4)$$

where d_i is any given diameter class, and D_{dom} the mean dominant diameter.² Thus a dominant tree will have $r = 1$. An emergent above the general level of the canopy will have r greater than 1. The empirical relationship observed between dominance and increment is shown by Figure 2. Here tree increment data from

² Dominant diameter is the mean diameter of 100 thickest stems/ha.

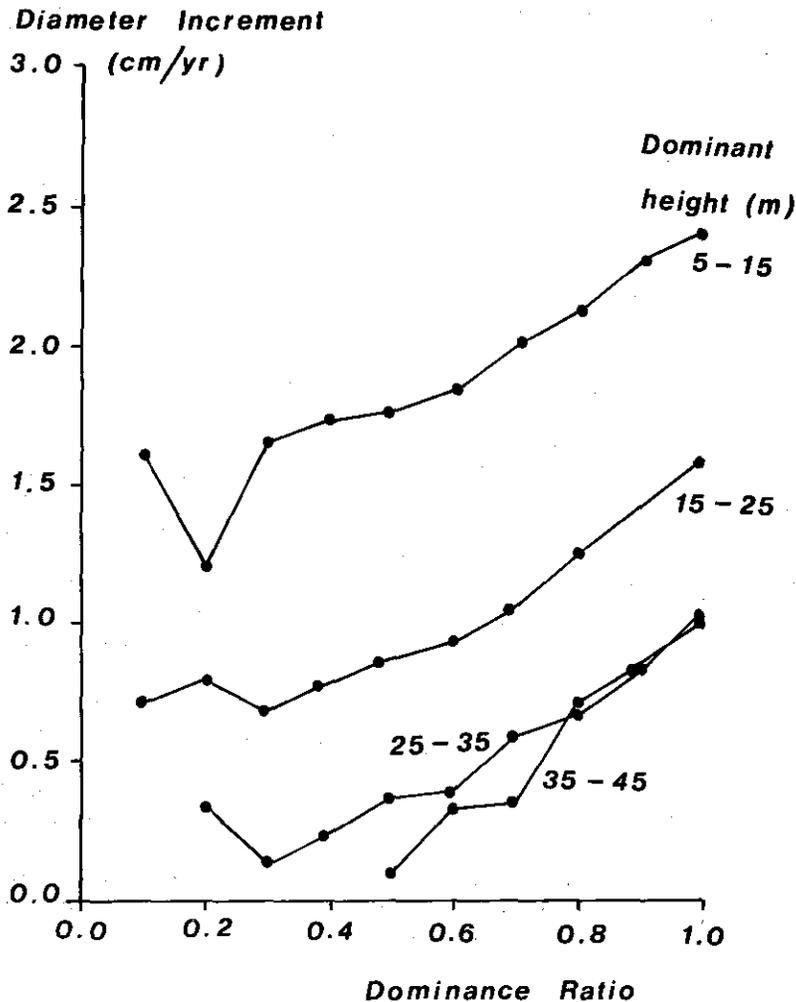


FIGURE 2. Mean diameter increment of cells defined by dominance ratio and dominant height classes for *Pinus patula*.

permanent sample plots were tabulated into classes defined by dominance and stand dominant height. Typically each point on the graph represents the mean of several hundred or thousand increment estimates.

Tree increment will also depend on age and site independently of stand density and tree dominance. This relationship was studied by tabulating the increment of dominant trees for classes of stand dominant height. The results are shown for *Pinus patula* permanent sample plots in Figure 3. A number of exponential decay functions can be fitted to this type of curve, of which the most suitable appeared to be

$$\Delta d_i = a + b_0 \cdot \exp[-b_1 \cdot H] \quad (5)$$

where Δd_i is diameter increment and a , b_0 , and b_1 are the fitted coefficients. This model was then extended to incorporate dominance r and stand relative basal area q . A number of extensions were studied, not only with reference to the goodness of fit to the increment data tabulated by H , r , and q classes, but also

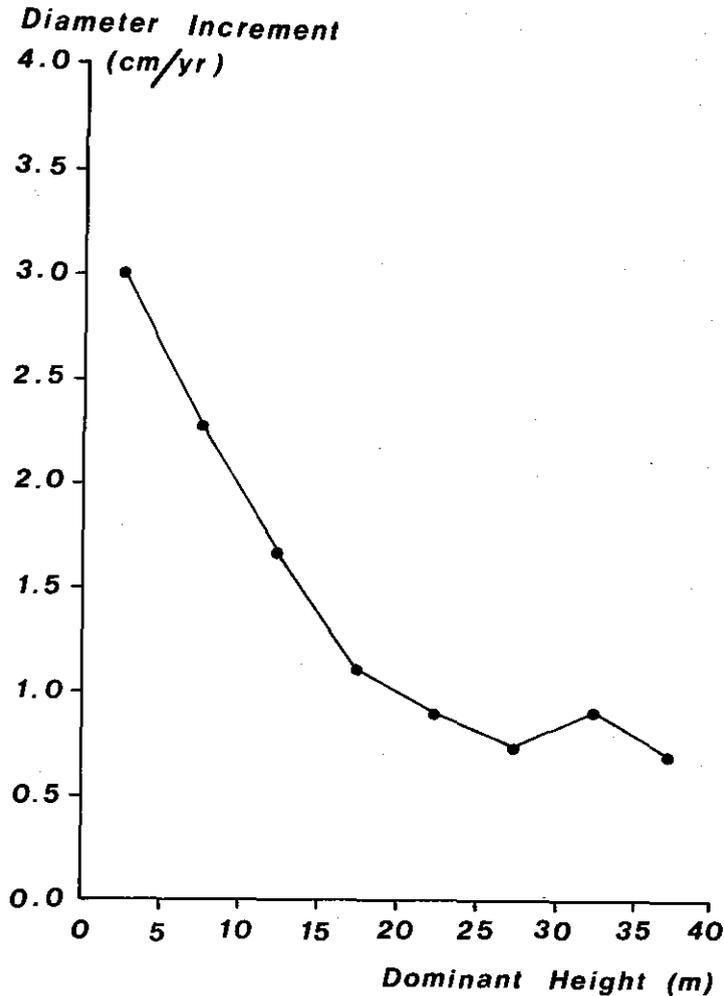


FIGURE 3. Mean diameter increment for dominant trees classified by stand dominant height for *Pinus patula*.

as they affected the behaviour of the model as a whole. The final choice was

$$\Delta d_i = a + b_0 \cdot \exp[H \cdot (b_1 + b_2 \cdot r + b_3 \cdot q + b_4 \cdot rq)] \quad (6)$$

The general behaviour of equation (6) can be understood qualitatively in terms of interacting growth processes. It is basically an exponential decay, from a value at $H = 0$ of $(a + b_0)$. The asymptotic minimum increment is given by a . The multiplier to the H in the main bracket controls the rate of decline of increment. It is a linear function of the dominance of the tree r and the relative basal area q and their interaction. From Table 2 it can be seen that the coefficients b_2 and b_3 are positive. The effect is that an increase in dominance or in relative growth potential both result in a decrease in the rate at which increment declines with H ; or, at a given H , an increase in either dominance or growth potential results in an increase in predicted increment. The negative interaction of rq (b_4 in Table 2) is due to the fact that at wider spacings (higher q) the same numerical change in r will denote a smaller real difference in competitive status than at a closer

TABLE 2. Coefficients for the diameter increment model (see equation (6) in text).

Species	a	b_0	b_1	b_2	b_3	b_4	N	R^2
<i>P. patula</i>	0.1818	3.499	-0.2306	0.1492	0.1478	-0.1112	1,663	0.54
<i>P. radiata</i>	0.0	1.008	-0.1532	0.1629	0.0948	-0.0854	1,834	0.59
Cypress	0.1812	2.936	-0.2054	0.1477	0.1100	-0.0894	1,438	0.60

Note: The parameters for the *P. patula* and cypress equations were fitted using iterative nonlinear least-squares estimation. This iterative method was unstable for the *P. radiata* data, for which the a parameter was poorly defined. In this case, a was assumed to be zero, and direct linear least-squares estimation used for fitting the other parameters. N is the number of tabulated data cells, not the number of raw tree increment observations. Tabulated means, rather than raw observations, were used in order to weight the regression in favour of the rarer plots at nonnormal stand densities; and to reduce the effect of the two level nesting (plot, tree) of increment data on the parameter estimates.

spacing. Thus the relative importance of dominance becomes less as the stand tends toward more open growing conditions. Table 2 gives the coefficients for equation (6) for the three species. The statistics given refer to the fit to tabulated values, and not to the raw data, for which the variances would be higher and the coefficient of multiple determination (R^2) lower.

In the simulation model, the application of equation (6) was modified by imposing two constraints. Firstly, the value of r was not permitted to exceed 1.2; or more precisely, if r was greater than 1.2, then 1.2 was used instead of the actual value for determining the increment of a given class. This limit of 1.2 for dominance was determined as the maximum observed in the data, with the exception of a few rare anomalies. Without this constraint, positive feedback could occur which results in excessively high increments being predicted. Secondly, q values for a stand were not permitted to become less than zero. Instead, after increment had been added to the vector D , the basal area G was compared with G_{\max} as estimated from the dominant height H at the end of the growth period. If G was greater than G_{\max} then the diameters were reduced by the multiplier $\sqrt{G_{\max}/G}$. Consequently, at high stockings the model may attain, but not exceed, the maximum basal area.

A general method of simulating the effect of thinnings on the vector of diameters D corresponding to the fixed cumulative probabilities P was required. As a first stage, the distribution of thinnings is defined in terms of two variables:

- l —The probability that any given stem in the stand will be left after thinning.
- p —The proportion of the total stocking smaller than any given stem.

The relationship between l and p can be determined empirically for different types of thinning. Some examples of the types of relationships that might be expected are illustrated in Figure 4. In low thinning, small stems have a high probability of removal, which declines progressively, until the largest stems are almost certain to be left. Row or mechanical thinning on the other hand gives any stem an equal probability of being removed. Crown thinning could produce various effects, depending on the way it is applied. In the example illustrated in Figure 4 it is envisaged that competitors to selected dominants are being removed, leaving a higher residual proportion of suppressed and dominant trees, and a relative depletion of the codominant and intermediate fractions.

The total proportion of the stocking prior to thinning that is left afterwards will be the integral of l with respect to p —in effect the sum of the probabilities of remaining for all the trees on the stand.

**Probability of a stem
remaining - l**

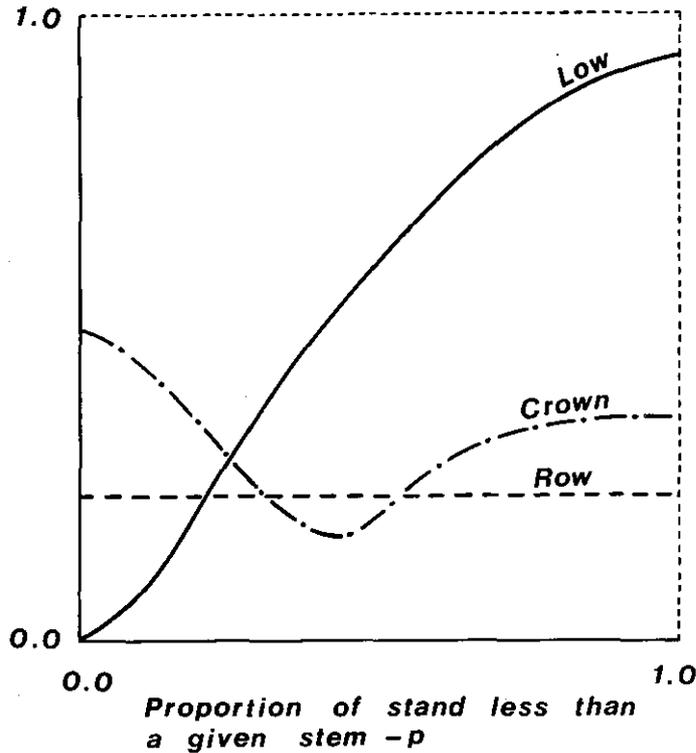


FIGURE 4. Some possible relationships between the probability of a tree remaining after thinning, and its diameter cumulative probability.

$$L = \int_0^1 l \cdot \partial p. \quad (7)$$

L is termed the leave fraction, and is the ratio of the stocking after thinning to the stocking before thinning.

Now consider the effect of a thinning upon the vector P . The proportion of the stand left after thinning up to a diameter d_i corresponding to a cumulative probability p_i will be

$$F(l) = \int_0^{p_i} l \cdot \partial p,$$

or in other words the sum of the probabilities of the removal of all trees up to this limit. $F(l)$ is the cumulative probability in terms of before thinning stocking corresponding to the diameter d_i . Expressing this as a proportion of stocking after thinning we scale by the leave fraction L —

$$p^*_i = 1/L \cdot \int_0^{p_i} l \cdot \partial p, \quad (8)$$

where p^*_i is the cumulative probability after thinning corresponding to the diameter d_i . Provided the function $l = f(p)$ is readily integrable, then equation (8) is easily applied to the vector P of before-thinning cumulative probabilities to

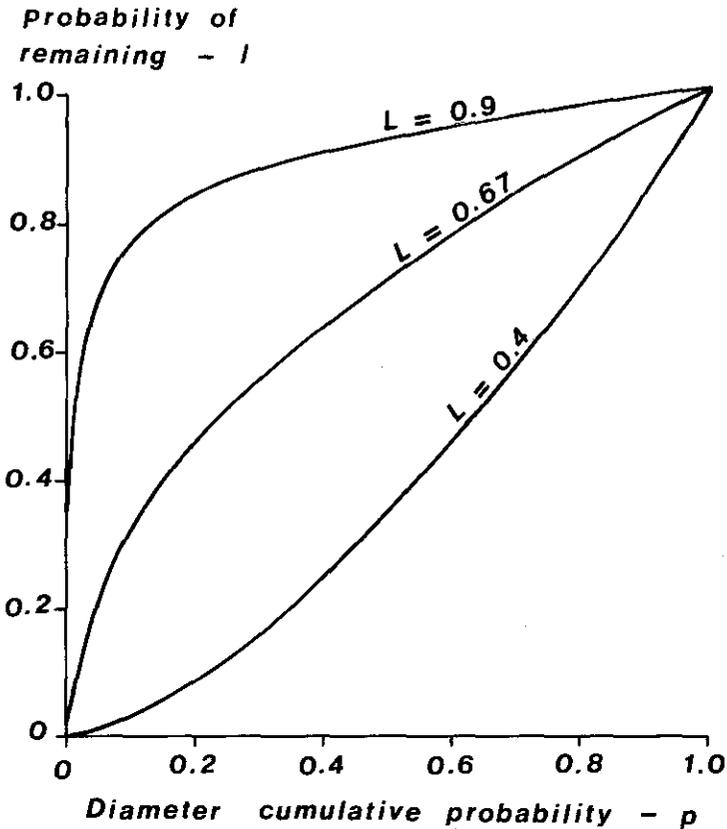


FIGURE 5. The effect of total leaf fraction L upon thinning distribution for the hypothetical low thinning regime represented by equation (9).

obtain a vector P^* of values after thinning. The vector D remains at this stage unchanged. In the simulation model, a new vector of diameters D^* corresponding to the standard probability intervals is generated from D and P^* using a linear interpolation procedure. This latter step is not of any theoretical interest, but does allow computer storage requirements to be greatly reduced when a large number of stands are being simulated simultaneously, since only the D vector need be retained for each stand. The P values are implicit in equation (1).

Empirical data concerning thinning distributions were not available at the time of this study, and instead certain assumptions were made in order to provide a useful model. Two basic types of thinning were simulated. The first is row or mechanical thinning, and the second is low selective thinning. It is self-evident that with row thinning (or any thinning that is unbiased or "neutral" with regard to stem size) the diameter distribution of the stand will not significantly change, and hence we would expect that

$$P^*_i = P_i.$$

This result can be derived from equations (7) and (8) by noting that

$$l \text{ is constant with respect to } p, \text{ say } l = c,$$

and hence

$$L = \int_0^1 c \cdot \partial p \quad \text{from equation (7)}$$

$$= c$$

and therefore

$$l = L.$$

Substituting into equation (8) we have

$$p_i^* = 1/L \cdot \int_0^{p_i} L \cdot \partial p$$

$$= p_i.$$

Thus with row thinning neither the diameter vector D nor the probability vector P is altered; only the total stocking changes.

For low thinnings, the distribution was assumed to follow a family of power curves described by the function

$$l = p^c. \quad (9)$$

These are illustrated for different leave fractions in Figure 5. In this distribution, a large tree is much more likely to remain after thinning than a small one. The degree of selectivity depends on the intensity of thinning. At low thinning intensities there is a high proportion of removals among the smallest stems, with very little effect on the larger fraction. At higher intensities, the distribution of thinnings tends to become more even. From equation (7) we have

$$L = \int_0^1 p^c \cdot \partial p,$$

and hence

$$L = 1/(c + 1);$$

thus

$$c = 1/L - 1$$

and the relationship (9) can be rewritten as

$$l = p^{1/L-1}. \quad (10)$$

Equation (8) then becomes, by substitution of (10) and integration

$$p_i^* = 1/L \cdot p_i^{(1/L-1)}. \quad (11)$$

Equation (11) is the function used to simulate the thinning treatments discussed in the next section.

RESULTS

Three types of validation were performed on the model. These were, firstly, the mass simulation of all permanent sample plots which contributed data to the model; secondly, the simulation of some spacing experiments which were geographically remote from the main sources of data, and whose results were not used during function development; and thirdly, some detailed studies of specific thinning cases.

The results of the mass simulation of permanent sample plots is shown in Table 3. In this case a single statistic was selected for comparison between model predictions and observed growth. Each simulation run was of one permanent

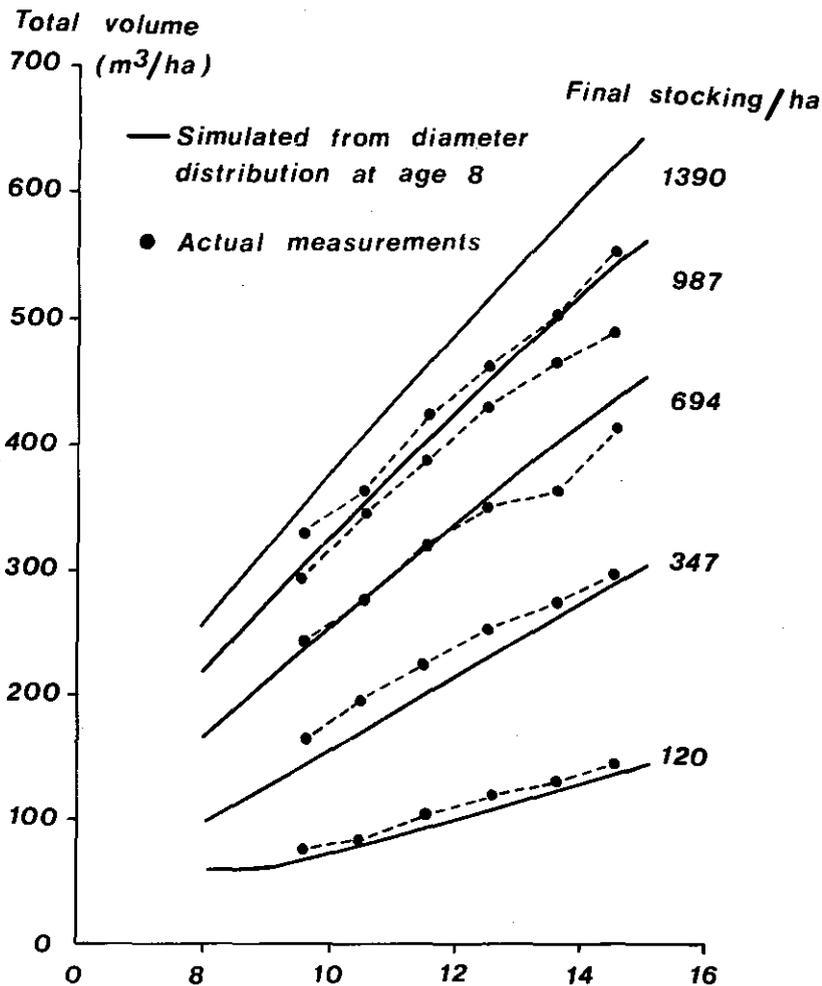


FIGURE 6a. Simulated and actual total volumes for two thinning experiments in Tanzania—Experiment 345, *Pinus patula*.

sample plot. The initial conditions of height and diameter distribution were those actually observed. Thinnings were defined in terms of leave fraction (i.e., stocking after thinning divided by stocking before thinning). The actual thinnings observed on each plot were used to set the thinning schedule for the simulator. The deviation of predicted total volume from observed total volume was computed for each remeasurement, and expressed as a percentage of the predicted volume. That is

$$E = (V_o - V_s)/V_s \%$$

where V_o and V_s are the observed and simulated total volumes respectively. E is the percentage residual. These residuals were tabulated by species and remeasurement number; the mean residual or net bias, and the standard deviation of the percentage residuals are the statistics shown for each species in Table 3. The number of plots involved for a given cell of the Table are also shown.

Table 3 shows that the bias in total volume prediction is negligible. The standard

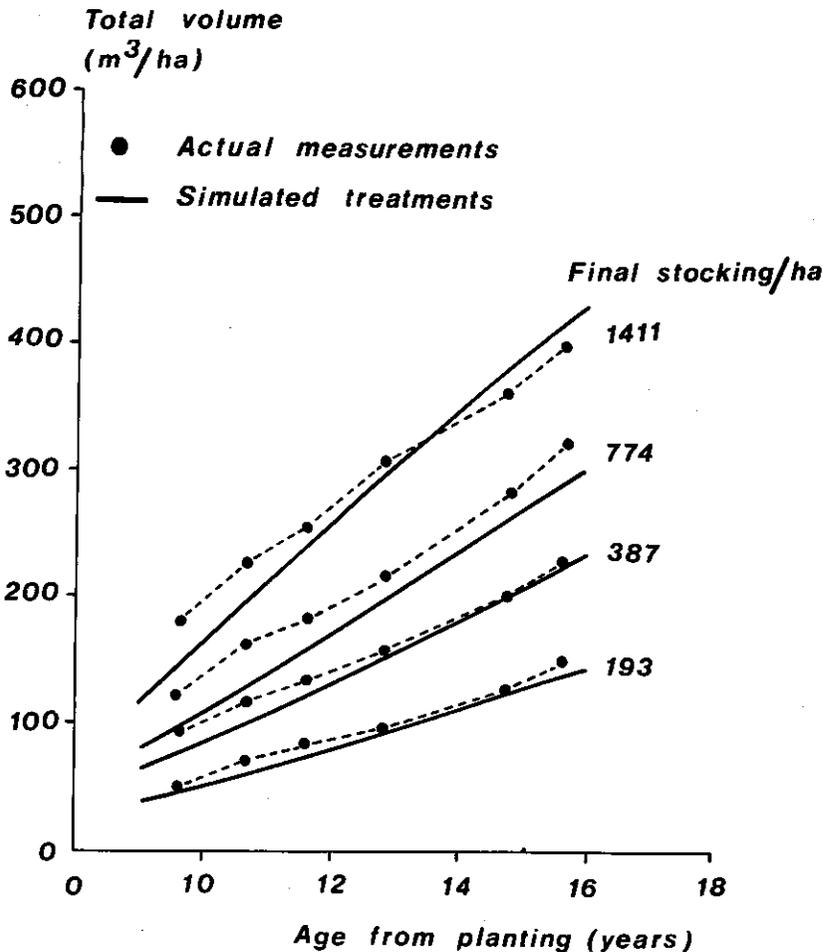


FIGURE 6b. Simulated and actual total volumes for two thinning experiments in Tanzania—Experiment 661, *Pinus patula*.

deviation of the percentage residuals is approximately 12 percent for cypress, 18 percent for *P. radiata*, and 19 percent for *P. patula*. This appears to be a similar order of precision to that obtained for these data with aggregate yield models of the type developed by Wanene (e.g., 1975, 1976). Given the magnification of bias which occurs in finite difference systems when they are integrated, this is a useful achievement. We may note, therefore, that the model is at least as accurate in predicting total volume as an aggregate stand model constructed with the same data; it is also of course much more flexible and provides considerably more detailed information about the stand.

Figures 6(a) and 6(b) show the results from simulations of two spacing experiments on *P. patula* in Tanzania. Both are designed in a similar manner, and involve establishment at approximately 1,600 stems/ha (or 900 stems/ha for the lowest stocking treatments) with frequent thinnings ahead of competition between the third and eighth years until the final stocking for that treatment is reached. Experiment 345 is a clinal design with 8 treatments; 3 intermediate treatments are

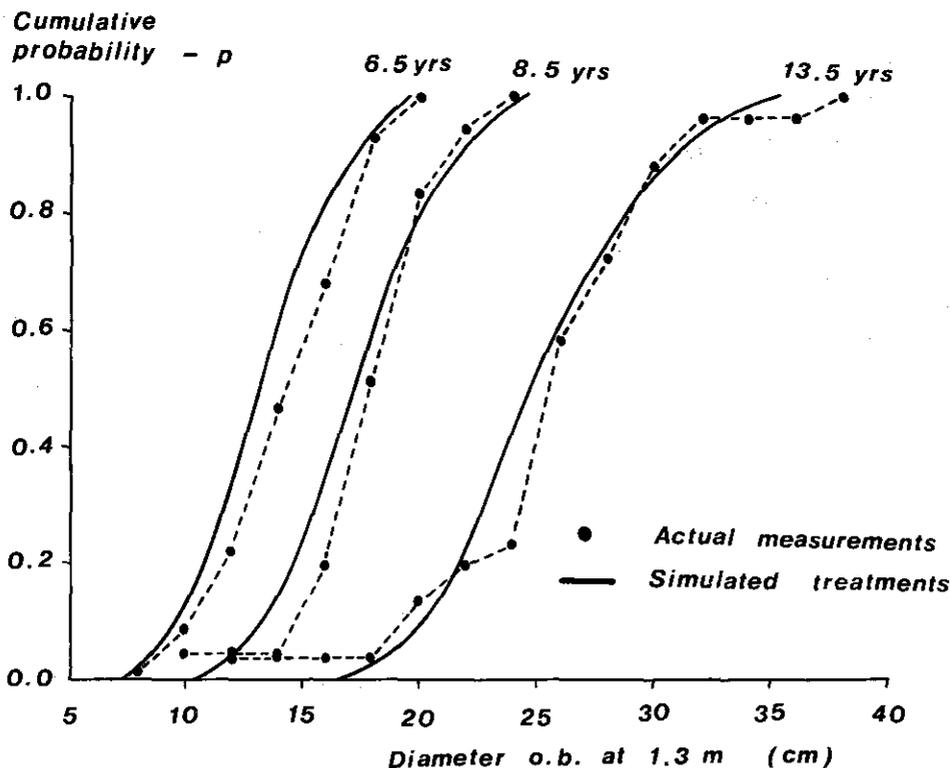


FIGURE 7. Simulation of a cypress permanent sample plot showing cumulative diameter distribution before first thinning and after first and second thinning.

omitted from Figure 6(a) for clarity. Experiment 661 is a randomized block design with 4 treatments. Further details are given in Adlard and Alder (1976).³

Both figures show that prediction of total volume by the simulator is quite accurate over a very wide range of competitive stress. The lowest stockings are essentially free of intertree competition, while the high density stands are close to the limiting basal area line. No systematic bias is apparent, with the possible exception of a tendency to produce too great an increment at the highest stocking. The importance of this particular validation is that it tests the model's response on a set of data that is independent from that used to develop the model, as well as covering a very wide range of spacing, and two distinct site indices (as can be seen, volume production on experiment 345 is considerably higher than that for experiment 661). Unfortunately, similar independent experimental data for the other two species was not available at the time of writing.

Figure 7 shows how the simulated diameter distribution responds to thinning. The data are from cypress permanent sample plot 279 in Kenya, which was initially stocked at 1,430 stems/ha, and subsequently thinned at 8 years down to 940 stems/ha and at 13 years to 520 stems/ha. This corresponds more or less to normal thinning treatment for sawlog stands in the area. Figure 7 shows the actual cumulative frequency distribution prior to first thinning, at 6.5 years, and shortly after first and second thinnings, at 8.5 and 13.5 years respectively. The simulation

³ Adlard, P. G., and D. Alder. 1976. The use of models for the evaluation of spacing and thinning effects in fast-growing conifers. Discussion paper for Group 2.02 XVth IUFRO World Congress, Oslo, 1976.

TABLE 3. Bias and standard deviation of percentage differences in total volume between observed and simulated permanent plots.

Remea- sure- ment no.	<i>Cupressus</i> sp.			<i>Pinus radiata</i>			<i>Pinus patula</i>		
	N	Bias	s.d.	N	Bias	s.d.	N	Bias	s.d.
		Percent	Percent		Percent	Percent		Percent	Percent
1	160	3.3	12.6	169	-1.0	9.9	166	2.7	16.4
2	152	0.7	10.4	169	2.9	14.9	162	4.5	18.7
3	148	0.7	11.5	159	5.4	16.8	128	5.7	20.1
4	143	0.6	10.3	157	3.9	19.0	122	4.2	18.0
5	127	0.1	12.1	155	2.8	18.5	116	5.8	17.5
6	116	0.0	12.7	155	4.2	19.6	112	5.0	17.2
7	103	1.3	12.2	127	2.2	21.1	105	3.1	16.2
8	95	2.1	14.9	122	0.5	20.5	102	1.0	17.8
9	88	1.0	16.5	103	0.3	19.5	100	0.8	19.1
10	73	1.7	16.7	72	0.2	20.7	92	-0.8	19.1
11	35	-7.4	13.0	56	1.0	20.7	54	-4.4	21.1
12	17	-13.1	15.2	30	3.0	18.7	14	11.7	32.8

Note: *N* is the number of plots contributing to a given estimate of bias and standard deviation. Bias percent is the mean of the differences between observed and simulated volumes, expressed as a percentage of the simulated volume. s.d. is the standard deviation of these percentage differences.

uses a site index and production class (i.e., initial basal area) estimated from the plot, and uses the same thinning history as the plot. However, the distribution of thinnings is based upon the assumed relation given in equations (9) and (11). It can be seen from Figure 7 that there is a close correspondence between the actual and the simulated distributions. However, prior to the first thinning the actual distribution is somewhat flatter than the simulated one; whilst at each of the thinnings it appears that a rather higher proportion of intermediate stems and a lower proportion of the most suppressed trees are removed in the actual thinning, as compared with the simulated one.

The model presented here appears to provide a sound basis for predicting yields from a wide variety of thinning regimes for the species studied. It is sufficiently compact and efficient to provide a means for direct updating and projection of forest inventory records. As such, it fulfills a major niche in growth and yield prediction for the tropical highland conifers of East Africa and Malawi.

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